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A robust Bayesian dynamic linear model for Latin-American economic time series: "the Mexico and Puerto Rico cases"

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Abstract The traditional time series methodology requires at least a preliminary transformation of the data to get stationarity. On the other hand, robust Bayesian dynamic models (RBDMs) do not assume a regular pattern or stability of the underlying system but can include points of statement breaks. In this paper we use RBDMs in order to account possible outliers and structural breaks in Latin-American economic time series. We work with important economic time series from Puerto Rico and Mexico. We show by using a random walk model how RBDMs can be applied for detecting historic changes in the economic inflation of Mexico. Also, we model the Consumer Price Index, the Economic Activity Index and the total number of employments economic time series in Puerto Rico using local linear trend and seasonal RBDMs with observational and states variances. The results illustrate how the model accounts the structural breaks for the historic recession periods in Puerto Rico.

Keywords Robust Bayesian dynamic model · Outliers and structural breaks · Latin-American time series · Consumer Price Index · Economic Activity Index · Total number of employments

JEL Classification C11 · C40 · G17 · N16

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1 Introduction

Economic Latin-American time series variables can be complex with high frequency data. From a frequentist perspective, techniques for fitting time series models use a preliminary data-set transformation in order to get stationarity and therefore important information about the dynamic system can be lost. On the other hand, from a Bayesian perspective the use of RBDMs with weakly-robust priors for the observation and state variances has been revolutionary in recent years due to flexibility for detecting outliers and structural breaks, the straightforward computational techniques based in Markov Chain Monte Carlo (MCMC) and the natural update from Bayes theorem without a preliminary data-set transformation.

From a frequentist perspective the study of time series with structural changes has been of far reaching in econometric theory for univariate time series, frequentist dynamic models, volatility and even financial return models. The seminal paper of Tsay (1988) considers least square techniques and residual variance ratios for detecting outliers, level shifts and variance changes in univariate time series. Hansen (2000) introduces a bootstrap method for detecting structural changes in regressors including structural shifts, polynomial trends and exogenous stochastic trends for frequentist dynamic econometric models. For volatility models, Vladimir (2009) uses local chance point analysis to intervals of homogeneity in order to account possible structural breaks. Fryzlewicz and Rao (2013) propose a method based in process transformation and binary segmentation for detecting multiple change points in auto-regressive conditional heteroscedastic models for financial returns.

From a Bayesian perspective in recent years RBDMs for detecting structural breaks have been proposed as an alternative to the usual Bayesian dynamic models. Ardia and Hoogerheide (2010) propose a Bayesian generalized autoregressive conditional heteroscedasticity dynamic model with Student-t innovations with applications to the R program (R Development Core Team 2015). Polson and Scott (2011) apply heavy-tailed priors in order to examine historical patterns of return on assets to financial time series. Fúquene et al. (2014) propose a new flexible class of heavy-tailed priors for detecting outliers and structural breaks in Bayesian dynamic linear models.

However, even though the qualities of RBDMs in the best of our knowledge there are no application of RBDMs to economic Latin-American variables for detecting their historic outliers and structural breaks. Therefore, in this work we use the RBDMs proposed by Fúquene et al. (2014) to modelling the historic change points in Latin-American economic variables from Mexico and Puerto Rico. We use this methodology because: (1) we can model a considerable variety of dynamic models: random walk, local linear trend and seasonal or a combination of those models with weakly-robust priors for the observation and state variances, (2) the computational schemes can be applied easily for practitioners and (3) using RBDMs allows us to have posterior inference for the parameters from a Bayesian perspective.

The paper is organized as follows: Section 2 shows the prior variances specification for the RBDM. In Sect. 3, we apply a random walk RBDM to an economic time series of Mexico for detecting the historic outliers and level changes in the inflation of this country. Section 4 shows local trend and stationary RBDMs for accounting the abrupt changes in the economic recession periods in Puerto Rico. Finally we have the conclusions in Sect. 5.

2 Model specification

The dynamic linear model (DLM) is specified (see West and Harrison 1997) by a Normal prior distribution for the p-dimensional state vector at $t = 0$ as follows:

$$\boldsymbol{\theta}_0 \sim N_p(m_0, C_0), \tag{1}$$

with the set of equations:

$$y_t = F_t \boldsymbol{\theta}_t + v_t \quad v_t \sim N_m(0, V_t), \tag{2}$$

$$\boldsymbol{\theta}_t = G_t \boldsymbol{\theta}_{t-1} + \omega_t \quad \omega_t \sim N_p(0, W_t), \tag{3}$$

with $t = 1 : T$ and where F_t and G_t are known matrices of order $p \times p$ and $m \times p$ respectively. With v_t and ω_t two independent Gaussian random vectors with mean zero and known variance V_t and W_t respectively. The observation equation and state equation are (2) and (3), respectively. A set of prior distributions for the observation and state variances may be considered in practice. For example in order to have closed form full conditionals we could use gamma prior densities. However, in the presence of highly frequency data heavy-tailed priors are the best alternative. The scaled Beta2 prior for the precision $\lambda = 1/\tau^2$ is proposed in Fúquene et al. (2014) for modelling the variances (and precisions) in DLMs and defined as follows:

$$\pi(\lambda) = \frac{\Gamma(q+p)}{\Gamma(q)\Gamma(p)} \beta \frac{(\beta\lambda)^{q-1}}{(1+\beta\lambda)^{p+q}}; \quad \lambda > 0 \tag{4}$$

where β is the scale parameter. This paper considers the Student-t density coupled with a scaled Beta2 for modelling the observation and state errors (as in Fúquene et al. 2014) in Latin-American economic time series from Mexico and Puerto Rico. So, let $\theta \sim$ be a Student-t (μ, τ, υ) where υ are the degrees of freedom, μ the location and τ the scale of the Student-t density:

$$\pi(\theta|\tau^2) = \frac{k_1}{\tau}\left(1 + \frac{1}{\upsilon}\left(\frac{\theta-\mu}{\tau}\right)^2\right)^{-(\upsilon+1)/2}, \quad \upsilon > 0, -\infty < \mu < \infty, -\infty < \theta < \infty, \tag{5}$$

where $k_1 = \dfrac{\Gamma((\upsilon+1)/2)}{\Gamma(\upsilon/2)\sqrt{\upsilon\pi}}$. We have that $\pi(\theta) = \int_0^\infty \pi(\theta|\tau^2)\pi(\tau^2)d\tau^2$ and therefore the marginal prior as follows:

$$\pi(\theta)=\begin{cases}\beta^q v/(\theta-\mu)^{q+1/2}2F1(p+q,q+1/2,(\upsilon+1)/2+p+q,1-\beta v/(\theta-\mu)^2) & \text{if } \theta\neq\mu,\\ k_1\mathrm{Be}(\mathrm{q}+1/2,\mathrm{p}+\mathrm{v}/2)/\mathrm{Be}(\mathrm{p},\mathrm{q}) & \text{if } \theta=\mu,\end{cases}$$

with $2F1(a, b, c, z)$ the hypergeometric function (see 15.1.1 of Abramowitz and Stegun 1970) and we have that $\pi(\theta)$ is the Student-t-Beta(υ,p,q,β) (see Fúquene et al. 2014 for the proof of this result). The variances of the RBDM are Student-t-Beta$(\upsilon,q,p,\frac{1}{\beta})$ densities (with the Beta2 prior for the precision as $\lambda=1/\tau^2$). Here $W_{t,i}$ denotes the ith diagonal element of $W_{t,i}$, $i=1,\ldots,n$ the hierarchical Student-t-Beta$(\upsilon,q,p,\frac{1}{\beta})$ prior can be summarized as follows:

$$\begin{aligned}
&V_t^{-1}=\lambda_y\omega_{y,t},\quad W_{t,i}^{-1}=\lambda_{\theta,i}\omega_{\theta,t_i},\\
&\lambda_y|q\sim\mathrm{Gamma}(\mathrm{q},(\beta\rho_{\mathrm{y}})^{-1}),\quad \lambda_{\theta,\mathrm{i}}|\mathrm{q}\sim\mathrm{Gamma}(\mathrm{q},(\beta\rho_{\theta,\mathrm{t_i}})^{-1}),\\
&\omega_{y,t}\sim\mathrm{Gamma}(\upsilon/2,2/\upsilon),\quad \omega_{\theta,\mathrm{t_i}}\sim\mathrm{Gamma}(\upsilon/2,2/\upsilon),\\
&\rho_y\sim\mathrm{Gamma}(\mathrm{p},1),\quad \rho_{\theta,\mathrm{t_i}}\sim\mathrm{Gamma}(\mathrm{p},1),
\end{aligned}$$

For each t, the posterior distribution of the latent variables $\omega_{y,t}$ and ω_{θ,t_i} is useful in order to account the outliers and abrupt changes in the economic time series. Values of $\omega_{y,t}$ and ω_{θ,t_i} smaller than one indicate possible outliers or abrupt changes respectively. A Gibbs sampler scheme can be implemented by using the full conditional in closed form of RBDMs (see Appendix 1).

2.1 Illustration RBDM with a toy example: the annual CPI from Puerto Rico

We consider now the annual Consumer Price Index (CPI) in Puerto in the log-scale in order to illustrate how a RBDM works. We use a local linear trend model (i.e., linear growth model) for fitting the trend and slope of the Consumer CPI in logarithm scale as follows:

$$\begin{aligned}
y_t&=\mu_t+v_t,\quad v_t\sim N(0,V_t),\\
\mu_t&=\mu_{t-1}+\xi_{t-1}+\omega_{t,1},\quad \omega_{t,1}\sim N(0,W_{t,1}),\\
\xi_t&=\xi_{t-1}+\omega_{t,2},\quad \omega_{t,2}\sim N(0,W_{t,2}),
\end{aligned}\tag{6}$$

with uncorrelated errors v_t, $\omega_{t,1}$ and $\omega_{t,2}$ and where

$$\theta_t=\begin{bmatrix}\mu_t\\ \xi_t\end{bmatrix},\quad G=\begin{bmatrix}1&1\\0&1\end{bmatrix},\quad W_t=\begin{bmatrix}\sigma^2_{\mu,t}&0\\0&\sigma^2_{\xi,t}\end{bmatrix},\quad F=\begin{bmatrix}1&0\end{bmatrix}.$$

For this toy example we use a Student-t-Beta2 where $p=q=1$, and $1/\beta=10{,}000$ as is proposed in Pericchi and Perez (2010). We have convergence of all parameters using 10,000 iterations after a burn-in phase period of 5000 iterations. Figure 1 displays how by using the RBDM an outlier in the year 2000 is obtained. The changes in the trend shows the level changes in 1990 and 2001 and the slope presents a change in 2003 (Fig. 1).

3 The case of Mexico

In this section we use a random walk RBDM in order to detect outliers and structural breaks of the inflation in Mexico. We use the monthly logarithm of the CPI-variations from 1696 to 1983 in order to accounting possible changes in inflation in Mexico during this period such as: (1) The monetary devaluation in 1976 and 1982. (2) The value-added tax imposed in 1980 with the posterior modification in 1983. (3) Some changes in the payment to employees and the increase in gasoline prices and (4) The modification in the economy in Mexico in 1983. The random walk model RBDM can be written as follows:

$$\begin{aligned} y_t &= \theta_t + v_t, \quad v_t \sim N(0, V_t), \\ \theta_t &= \theta_{t-1} + \omega_t, \quad \omega_{t,1} \sim N(0, W_t), \end{aligned} \tag{7}$$

with the prior specification showed in previous Section. We implement the Gibbs sampling scheme showed in Appendix 1 and from a visual assessment of the Gibbs output we have that convergence has been achieved in Appendix 2. The ergodic means are nonetheless pretty stable in the middle of the plots and the decay of the empirical autocorrelation function is very fast.

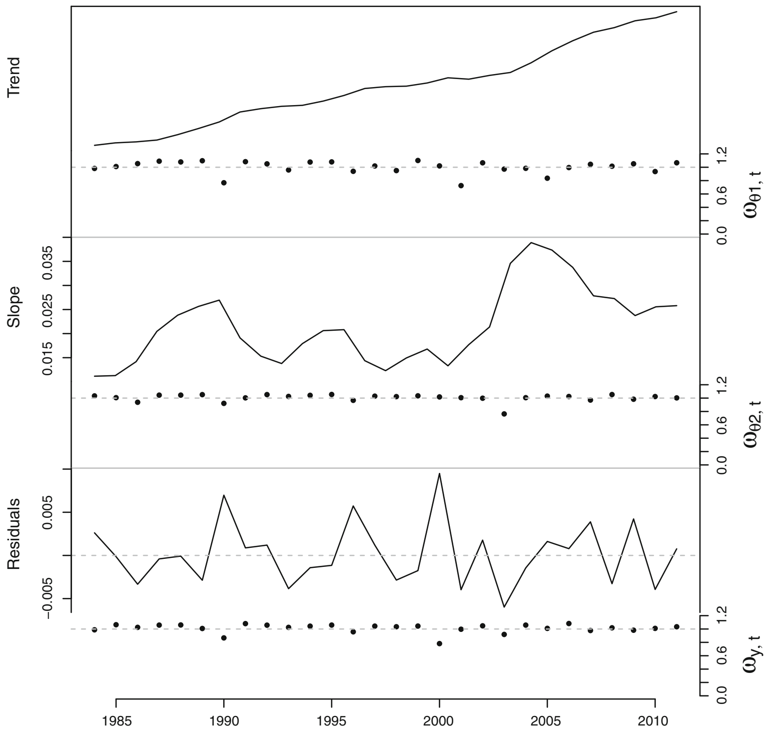

Fig. 1 Outliers and structural breaks in the annual logarithm Consumer Price Index in Puerto Rico using the robust approach. The *right* scale is for the latent $\omega_{y,t}$ and ω_{θ,t_i} parameters

According to Fig. 2. The state parameters show significant level changes in September 1976 and January 1982. These changes likely represent the exchange rate devaluation in September 1976 and just one month before of the second devaluation in 1982. On the other hand, in Table 1 the expectations for the latent parameters for identifying the outliers in the random walk RBDM are presented. The outliers in January 1980 and January 1983 probably showed the value-added tax imposed and consequently modified in those dates with also the modification in the economy in Mexico in 1983. The most dramatic increase in payment to employees could be exposed for the extreme values in January 1974, October 1976 and January 1979. The exchange rate devaluation presented in August 1982 and December 1982 are also presented in Table 1. Due to the increase in gasoline prices the RBDM could show outliers in December 1973, November 1974 and November 1976. Finally, we think this random walk RBDM could be useful not only to accounting changes in this economic time series but also may be useful as an pre-intervention dynamic model.

4 The case of Puerto Rico

In this section we use the linear trend RBDM presented in (6) and a linear trend RBDM with a seasonal component in order to model the logarithm of CPI, AEI and total number of employments (TNE) from January 1980 to December 2012 in Puerto Rico. The CPI and Economic Activity Index (EAI) are two economic

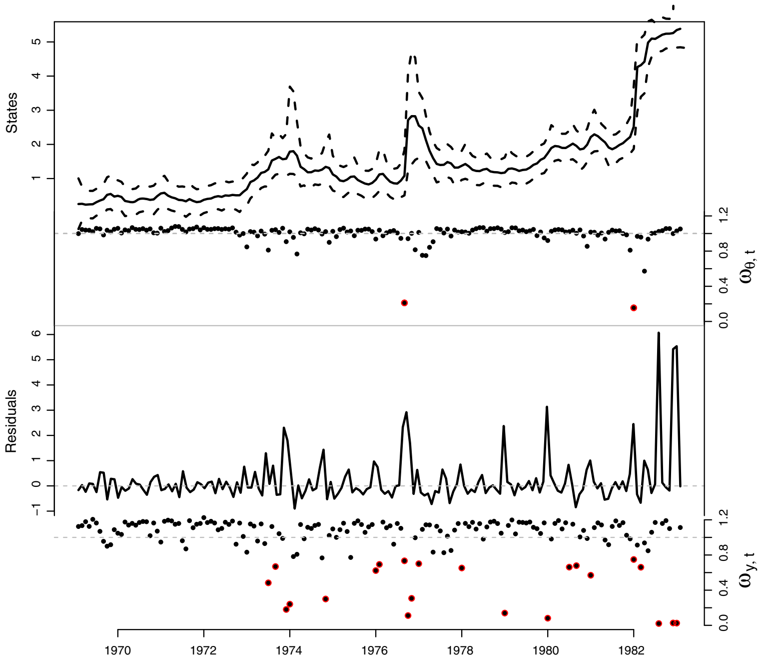

Fig. 2 Outliers and structural breaks in CPI-variations from Mexico 1969–1983. The *right* scale is for the latent $\omega_{y,t}$ and ω_{θ,t_i} parameters. *Red points* illustrate the outliers and structural breaks (color figure online)

Table 1 Posterior mean of $\omega_{y,t}$ for the monthly CPI-variations from Mexico 1969–1983

Month/year	$E(\omega_{y,t}\|y_{1:T})$
Jul 1973	0.8118393
Dec 1973	0.18126535
Jan 1974	0.24155247
Nov 1974	0.30013551
Oct 1976	0.11182539
Nov 1976	0.30769569
Jan 1979	0.14002184
Jan 1980	0.08193927
Aug 1982	0.02124920
Dec 1982	0.02747282
Jan 1983	0.02584617

indexes widely used for describing the economic situation of Puerto Rico. The CPI and EAI are useful for accounting the inflation through of price fluctuations and the real economic activity. EAI and TNE are very correlated in the sense that EAI is computed by using also TNE. However TNE is an interesting time series for the quarterly seasonal component and also for the historic fact that in Puerto Rico in July 2009 near of 17,000 employments lost their jobs for the recent economic crisis. We find that by using RBDMs historical changes are detected as structural breaks in the trend of the models.

Now we describe briefly some of the important historical changes useful for the interpretation of the results.

4.1 CPI historical changes

A first index for accounting the inflation in Puerto Rico was born by using the cost of living for working families in 1940. The CPI was born in 1977 by including in the first index information on urban families, self-employed and the pensioners in Puerto Rico. Using addition products, in 1990 a few adjustments to items and services of CPI were proposed. A new study of Income and Expenses was made in the years 1999–2003 with a major change in the CPI methodology in March 2010. Currently, the basket of goods for CPI has the following major groups: food and beverages, housing, apparel, transportation, medical care, entertainment, education and communication and those groups are similar to the United States basket (see for example Department of labor and human resources 2008).

4.2 EAI historical changes

The monthly EAI includes the behavior of four economic indicators: total number of employments (thousands), cement sales (million bags), fuel consumption (millions of gallons) and electricity generation (million KWH). According to the Government Development Bank for Puerto Rico (2012) the EAI has a strong linear correlation of

0.97 with the gross national product (GNP). So, in this work we use EAI as one of the indicators useful for detecting recession periods in the economy of Puerto Rico during the last 35 years have been: 1980–1982, 1990–1991, 2001–2002 and since 2006.

Due to the quarterly seasonal component in TNE (see Fig. 3) we use a local trend with a seasonal component in this economic variable. The observation and system matrices of this model are:

$$F = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \end{bmatrix} \quad G = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & -1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix},$$

and the unknown parameters are the observations variance V_t and three elements for W_t:

$$W_t = \begin{bmatrix} \sigma^2_{\mu,t}, & \sigma^2_{\xi,t}, & \sigma^2_{s,t}, & 0, & 0 \end{bmatrix}$$

where $\sigma^2_{\mu,t}$, $\sigma^2_{\xi,t}$ and $\sigma^2_{s,t}$ are the unknown variances of the level of the series, the slope of the linear trend and the seasonal respectively.

Figure 4 displays the results by using CPI for the period of January 1980–December 2012. The residuals in bottom of Fig. 4 are given by $\hat{\epsilon}_t = y_t - E(F\theta_t|y_{1:T})$. By looking at the residuals there are no outliers. The slope is dynamically changing with only a sudden jump within the first recession period in September 1981.

Table 2 show how the trend has different jumps, the most dramatic one in September 2005 with $E(\omega_{\theta,t_1}|y_{1:T}) = 0.07$ and some other abrupt changes in the precedent years. Even though the CPI is an indicator of inflation, this dramatic change could have been an "alarm" for the economic recession Puerto Rico is

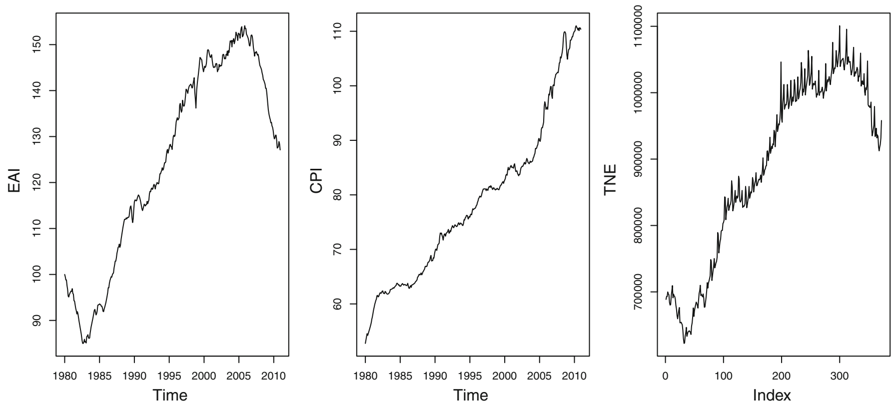

Fig. 3 Economic time series from Puerto Rico: EAI, CPI and TNE

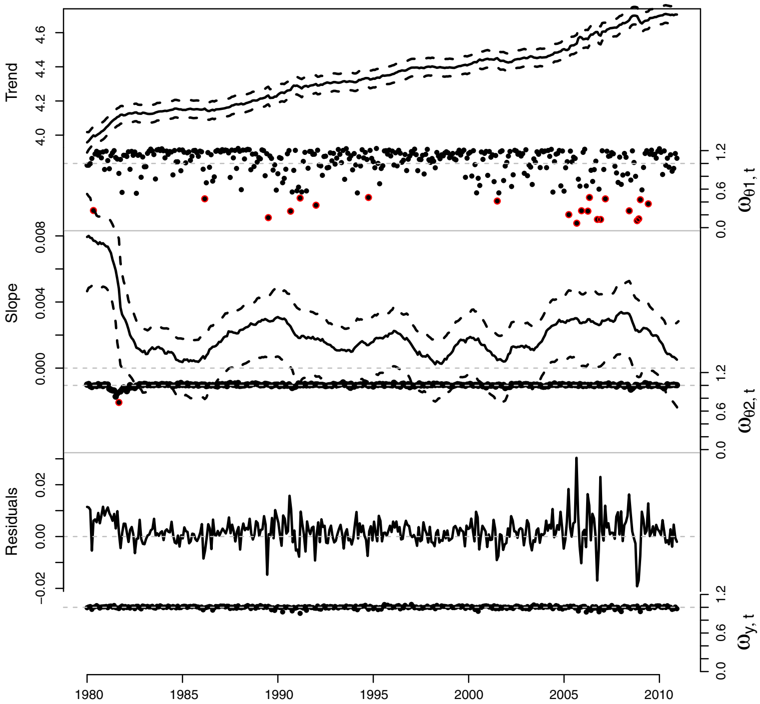

Fig. 4 Outliers and structural breaks in the logarithm monthly Consumer Price Index in Puerto Rico. The *right* scale is for the latent $\omega_{y,t}$ and ω_{θ,t_i} parameters. *Red points* illustrate the outliers and structural breaks (color figure online)

facing since 2006. Other dramatic changes are found in May 1980, July 1989 and September 1990, all of them within of recession periods in Puerto Rico.

The structural break in 1980 likely is due to changes in the methodology for computing the CPI. The change in the trend of the CPI at the years 1989 and 1990 could also be related to the implementation of the "Joint Committee on Taxation" in the United States, and its effect on the island. Figure 5 shows the abrupt changes in the trend for the EAI.

In particular, two dramatic level changes are presented in September 1989 and December 1989 in the beginning of the recession period of 1990. On the other hand other structural breaks are showed in July 1996, September 1998, December 1998 and July 2005 probably related with the rest of recession periods.

Figure 6 illustrates the trend, slope and seasonal posterior component with their corresponding credible intervals for TNE. An interesting feature by using RBDMs is that the credible intervals are not of constant width. For the seasonal component the 95 % interval is wider for the structural changes. Figure 7 displays the relationship between the time series for the recession periods. The abrupt changes in both indexes during the last period may be the consequence of the economic crisis that Puerto Rico has been suffering since 2006. In the recent economic crisis in Puerto

Table 2 Posterior mean of ω_{θ,t_1} for the monthly logarithm of the CPI, EAI and TNE

Month/year	$E(\omega_{\theta,t_1}\|y_{1:T})$-CPI
May 1980	0.26805231
Mar 1986	0.45021372
Jul 1989	0.15716954
Sep 1990	0.25732717
Mar 1991	0.46168657
Jan 1992	0.35024676
Oct 1994	0.47112292
Jul 2001	0.41570839
Apr 2005	0.20405587
Sep 2005	0.07041677
Dec 2005	0.26448903
Apr 2006	0.25815601
May 2006	0.47095720
Dec 2006	0.12863052
Jun 2008	0.26322651
Nov 2008	0.10991814
Dec 2008	0.13796592
Jan 2009	0.43613370
Jun 2009	0.37176951
	$E(\omega_{\theta,t_1}\|y_{1:T})$-EAI
Jul 1980	0.4805507
Mar 1983	0.3128145
Dec 1987	0.4969033
Sep 1989	0.2555551
Dec 1989	0.2868322
Jan 1990	0.4662116
Jul 1996	0.2527494
Dec 1996	0.3596205
Sep 1998	0.1603856
Oct 1998	0.3832146
Nov 1998	0.2897125
Dec 1998	0.2202965
Dec 2001	0.4586847
Jul 2005	0.3384897
	$E(\omega_{\theta,t_1}\|y_{1:T})$-TNE
Aug 1989	0.4428376
Aug 1990	0.4871985
Jul 2009	0.4016160

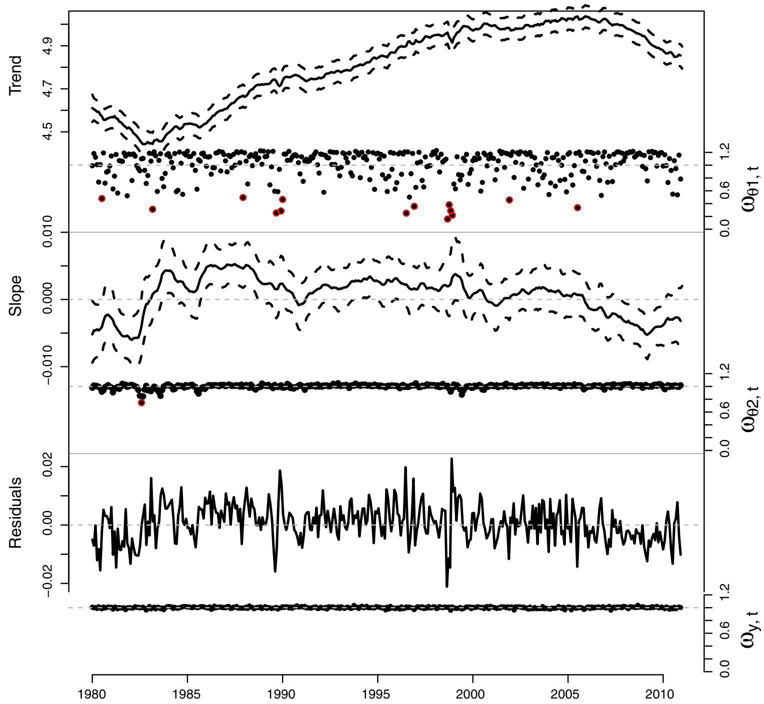

Fig. 5 Outliers and structural breaks in the logarithm monthly Economic Activity Index in Puerto Rico. The *right* scale is for the latent $\omega_{y,t}$ and ω_{θ,t_i} parameters. *Red points* illustrate the outliers and structural breaks (color figure online)

Rico 7816 public employments lost their jobs in July 2009 and the RBDM detects this change in the bottom of Fig. 7.

5 Conclusions

In this paper we apply Robust Bayesian Dynamic Models (RMBDs) to Latin-American time series from Mexico and Puerto Rico. The classes of RMBDs presented in this work with weakly-robust priors for the observation and state variances consider most of the empirical models used in the classical econometrics literature as random walk, linear trend, seasonal and a combination of those models. We found that using RMBDs allow us to account historic outliers and structural breaks in the inflation in Mexico and the economic recession periods in Puerto Rico. In fact, the structural changes have a contextual historical and economical meaning. Also, the model has the feature of producing not constant credible intervals over time even after accounting for boundary effects in the Latin-American variables. Finally we consider that in a future work new RMBDs could be implement to time series with weakly-shrinkage priors for the observation and state variances as is the

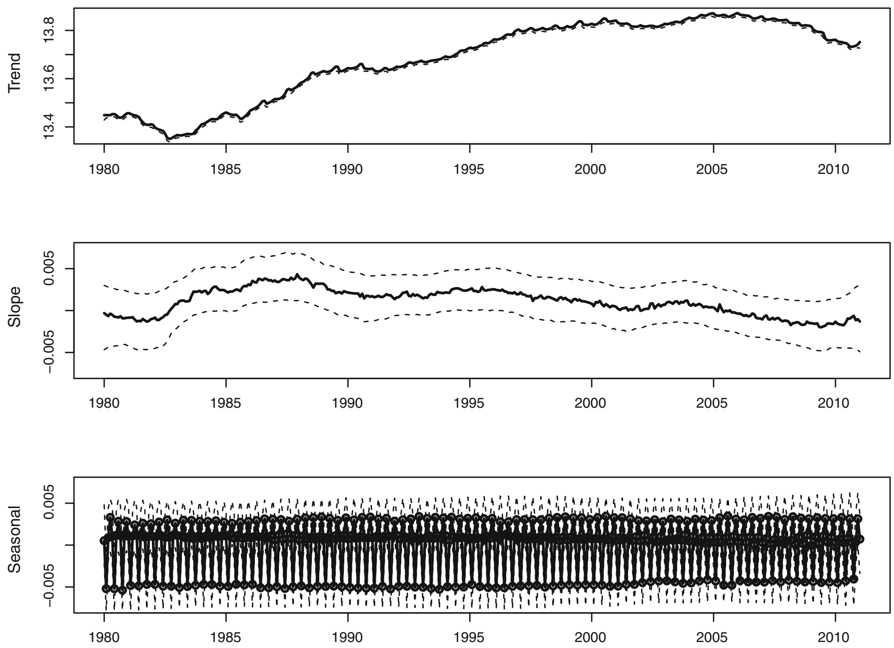

Fig. 6 Trend, slope and seasonal posterior mean parameters with their corresponding credible interval for TNE

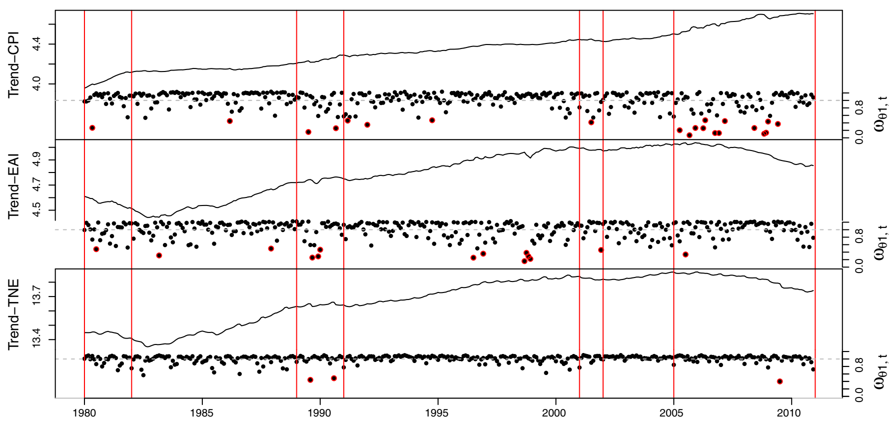

Fig. 7 Comparison structural breaks for the monthly Consumer Price and Economic Indexes in Puerto Rico. The *right* scale is for the latent ω_{θ,t_1} parameter. *Red points* illustrate the outliers and structural breaks. *Red lines* divide the recession time periods (color figure online)

case of generalized autoregressive conditional heteroscedasticity (GARCH) models for dynamic volatility approaches in finance.

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Appendix 1: Prior distributions and Markov Chain Monte Carlo algorithm

The scaled Beta2 distribution can be defined as a scale mixture of Gammas for the square of the scale as follows:

$$\tau^2 \sim \mathrm{Gamma}(\mathrm{p}, \beta/\rho) \tag{8}$$

$$\rho \sim \mathrm{Gamma}(\mathrm{q}, 1) \tag{9}$$

where Gamma(a, b) denotes the Gamma distribution:

$$p(x|\alpha, b) = \frac{1}{\Gamma(\alpha)\beta^{\alpha}} x^{\alpha-1} \exp\{-x/\beta\} \quad a > 0, b > 0, \tag{10}$$

with β the scale parameter. Therefore the scaled Beta2 prior for the square scale is the following:

$$\pi(\tau^2) = \frac{\Gamma(p+q)}{\Gamma(p)\Gamma(q)} \frac{1}{\beta} \frac{\left(\frac{\tau^2}{\beta}\right)^{p-1}}{\left(1 + \frac{\tau^2}{\beta}\right)^{p+q}}. \quad \tau > 0. \tag{11}$$

For precisions $\lambda = 1/\tau^2$, we assign the scaled Beta2 as

$$\pi(\lambda) = \frac{\Gamma(q+p)}{\Gamma(q)\Gamma(p)} \beta \frac{(\beta\lambda)^{q-1}}{(1+\beta\lambda)^{p+q}}; \quad \lambda > 0, \tag{12}$$

typically the hyper-parameters p, q are fairly small, for example $p = q = 1$, and β quite small (see Pericchi and Perez 2010) obtaining a bounded density at the origin, flat tails and a vague prior distribution.

The RBDMs can be written in hierarchical form as follows:

$$\begin{aligned} y_t &= F_t\boldsymbol{\theta}_t + v_t \quad v_t \sim N_m(0, V_t), \\ \boldsymbol{\theta}_t &= G_t\boldsymbol{\theta}_{t-1} + \omega_t \quad \omega_t \sim N_p(0, W_t), \end{aligned} \tag{13}$$

where the observation and state variances are given by:

$$\begin{aligned} & V_t^{-1} = \lambda_y \omega_{y,t}, \quad W_{t,i}^{-1} = \lambda_{\theta,i}\omega_{\theta,t_i}, \\ & \lambda_y | q \sim \text{Gamma}(\text{q}, (\beta\rho_{\text{y}})^{-1}), \quad \lambda_{\theta,\text{i}}|\text{q} \sim \text{Gamma}(\text{q}, (\beta\rho_{\theta,\text{t}_\text{i}})^{-1}), \\ & \omega_{y,t} \sim \text{Gamma}(\upsilon/2, 2/\upsilon), \quad \omega_{\theta,\text{t}_\text{i}} \sim \text{Gamma}(\upsilon/2, 2/\upsilon), \\ & \rho_y \sim \text{Gamma}(\text{p}, 1), \quad \rho_{\theta,\text{t}_\text{i}} \sim \text{Gamma}(\text{p}, 1), \end{aligned}$$

In order to obtain posterior inference on the state parameters $\boldsymbol{\theta}_1, \ldots, \boldsymbol{\theta}_T$, we use the forward filtering backward sampling (FFBS) given in Fruwirth-Schnatter (1994) which is practically a simulation of the smoothing recursions. The FFBS works as follows:

1. Use the Kalman Filter equations for (5). Let m_0 and C_0 (known) with $(\boldsymbol{\theta}_0|D_0) \sim N(m_0, C_0)$ and

$$\boldsymbol{\theta}_t | y_{1:t-1} \sim N(m_{t-1}, C_{t-1}) \tag{14}$$

   - The one step predictive distribution of $\boldsymbol{\theta}_t$ given $y_{1:t-1}$ is Gaussian $(\boldsymbol{\theta}_t|D_{t-1}) \sim N(a_t, R_t)$ with parameters:

$$a_t = G_t m_{t-1}; \quad R_t = G_t C_{t-1} G_t^{'}. \tag{15}$$

   - The one step predictive distribution of yt given $y_{1:t-1}$ is Gaussian $(y_t|D_{t-1}) \sim N(f_t, Q_t)$ with parameters:

$$f_t = F_t^{'} a_t; \quad Q_t = F_t^{'} R_t F_t + V_t.$$

   - The filtering distribution of $\boldsymbol{\theta}_t$ given $y_{1:t-1}$ is Gaussian $(\boldsymbol{\theta}_t|D_t) \sim N(m_t, C_t)$ with parameters:

$$m_t = a_t + A_t e_t; \quad C_t = R_t - A_t Q_t A_t^{'} \tag{16}$$

   where $A_t = R_t F_t Q_t^{-1}$, and $e_t = y_t - f_t$.

2. At time $t = T$ sample $\boldsymbol{\theta}_T$ from $N(\boldsymbol{\theta}_T|m_t, C_t)$.
3. For $t = (T-1) : 0$ sample $\boldsymbol{\theta}_t$ from $N(\boldsymbol{\theta}_t|m_t^*, C_t^*)$ with

$$m_t^* = m_t + B_t(\boldsymbol{\theta}_{t+1} - a_{t+1}) \quad C_t^* = C_t - B_t R_{t+1} B_t^{'}$$

   where $B_t = C_t G_{t+1}^{'} R_{t+1}^{-1}$.

In order to obtain now posterior inference for the rest of parameters in the observation and state variances we use the standard approach by considering the full conditional distribution proportional to the joint distribution of all random variables (parameters) considered. So, for example using (13) the full conditional for λ_y is given by:

$$\pi(\lambda_y|\ldots) \propto \prod_{t=1}^{T} \lambda_y^{1/2} \exp\left\{-\frac{\lambda_y \omega_{y,t}}{2}(y_t - F_t\boldsymbol{\theta}_t)^2\right\} \cdot \lambda_y^{q-1} \exp\{-\beta\rho_y\lambda_y\}, \tag{17}$$

hence,

$$\lambda_y|\ldots \sim \mathrm{Gamma}\left(\mathrm{q} + \frac{\mathrm{T}}{2}, \frac{1}{2}\mathrm{SSy}^* + \beta\rho_{\mathrm{y}}\right) \tag{18}$$

where $SSy^* = \sum_{t=1}^{T} \omega_{y,t}(y_t - F_t\boldsymbol{\theta}_t)^2$. The rest of full conditional distributions are given by:

$$\lambda_y|\ldots \sim \mathrm{Gamma}\left(\mathrm{q} + \frac{\mathrm{T}}{2}, \frac{1}{2}\mathrm{SSy}^* + \beta\rho_{\mathrm{y}}\right),$$
$$\lambda_{\theta,i}|\ldots \sim \mathrm{Gamma}\left(\mathrm{q} + \frac{\mathrm{T}}{2}, \frac{1}{2}\mathrm{SS}^*_{\theta,\mathrm{i}} + \beta\rho_{\theta,\mathrm{t_i}}\right)$$

where $SS^*_{\theta,i} = \sum_{t=1}^{T} \omega_{\theta,t_i}(\theta_{t_i} - (G_t\theta_{t-1})_i)^2$ for $i = 1, 2, \ldots, p$;

$$\omega_{y,t}|\ldots \sim \mathrm{Gamma}\left(\frac{\upsilon+1}{2}, \frac{\upsilon + \lambda_{\mathrm{y}}(\mathrm{y_t} - \mathrm{F_t}\boldsymbol{\theta}_{\mathrm{t}})^2}{2}\right),$$
$$\omega_{\theta,t_i}|\ldots \sim \mathrm{Gamma}\left(\frac{\upsilon+1}{2}, \frac{\upsilon + \lambda_{\mathrm{y}}(\theta_{\mathrm{t_i}} - \lambda_{\theta,\mathrm{i}}(\mathrm{G_t}\theta_{\mathrm{t-1}})_{\mathrm{i}})^2}{2}\right)$$

$$\rho_y|\ldots \sim \mathrm{Gamma}(\mathrm{p} + \mathrm{q}, \beta\lambda_{\mathrm{y}} + 1), \quad \rho_{\theta,\mathrm{t_i}}|\ldots \sim \mathrm{Gamma}(\mathrm{p} + \mathrm{q}, \beta\lambda_{\theta,\mathrm{i}} + 1),$$

Appendix 2: Convergence of parameters for the Mexico case

See Fig. 8.

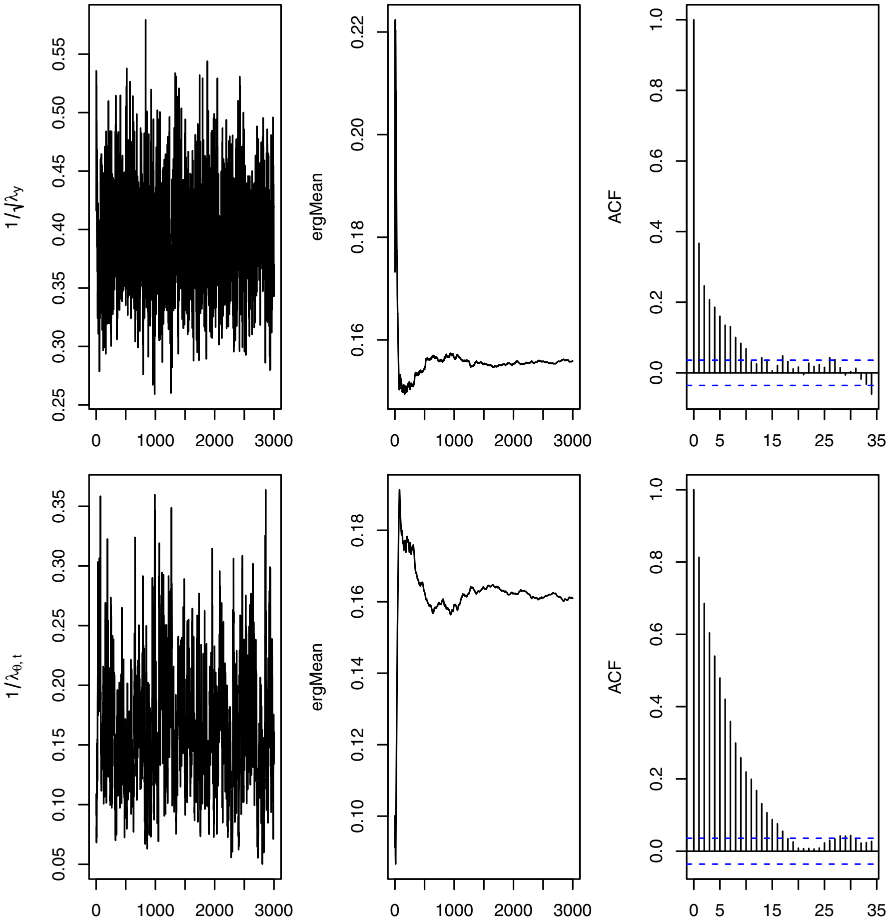

Fig. 8 Convergence diagnostic plots for the constant precision parameters of CPI-variations from Mexico 1969–1983. *Left* histograms for the precisions. *Middle* ergodic mean. *Right* autocorrelation plots

References

Abramowitz M, Stegun I (1970) Handbook of mathematical functions. In: National Bureau of standards, vol 46, Applied Mathematics Series

Ardia D, Hoogerheide L (2010) Bayesian estimation of the garch(1,1) model with student-t innovations. R J 2(2):41–47

Department of labor and human resources (2008) Study of revenue and expenditure of the urban consumer of Puerto Rico 1999–2003. Puerto Rico

Fruwirth-Schnatter S (1994) Data augmentation and dynamic linear models. J Time Ser Anal 15:183–202

Fryzlewicz P, Rao SS (2013) Multiple-change-point detection for auto-regressive conditional heteroscedastic processes. J R Stat Soc Ser B 76–5:903–924

Fúquene JA, Pérez M, Pericchi LR (2014) An alternative to the inverted gamma for the variances to modelling outliers and structural breaks in dynamic models. Braz J Probab Stat 28–2:288–299

Government Development Bank for Puerto Rico (2012) Economic activity index for the month of June 2012. Economy, Puerto Rico. http://www.gdb-pur.com/economy/documents/2012-Jun-GDB-EAIvFINAL-GS.pdf

Hansen BE (2000) Testing for structural change in conditional models. J Econom 97:93–115
Pericchi LR, Perez ME (2010) Limiting the shrinkage for the exceptional by objective robust bayesian analysis. Technical Report Centro de Bioestadstica y Bioinformtica, pp 1–17. http://repositorio.upr.edu
Polson NG, Scott JG (2011) Good, great or lucky? Screening for firms with sustained superior performance using heavy-tailed priors. Ann Appl Stat 6–1:161–185
R Development Core Team (2015) R: A language and environment for statistical computing. R Foundation for Statistical Computing. Vienna, Austria. http://www.R-project.org/
Tsay RS (1988) Outliers, leveel shifts, and variances changes in time series. Forescasting 7:1–20
Vladimir S (2009) Multiscale local change point detection with applications to value-at-risk. Ann Stat 37:1405–1436
West M, Harrison PJ (1997) Bayesian forecasting and dynamic models. Springer, Berlin